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# **mgwr Documentation**

***Release 2.0.1***

**pysal developers**

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mgwr is a Python implementation of multiscale geographically weighted regression for investigating process spatial heterogeneity and scale. It incorporates the widely used approach to modeling process spatial heterogeneity - Geographically Weighted Regression (GWR) as well as the newly proposed approach - Multiscale GWR (MGWR) which relaxes the assumption that all of the processes being modeled operate at the same spatial scale. Inferences are available for both approaches.



mgwr supports python 3.5 and 3.6 only. Please make sure that you are operating in a python 3 environment.

### 1.1 Installing released version

mgwr is available on the [Python Package Index](#). Therefore, you can either install directly with *pip* from the command line:

```
pip install -U mgwr
```

or download the source distribution (.tar.gz) and decompress it to your selected destination. Open a command shell and navigate to the decompressed folder. Type:

```
pip install .
```

### 1.2 Installing development version

Potentially, you might want to use the newest features in the development version of mgwr on github - [pysal/mgwr](#) while have not been incorporated in the Pypi released version. You can achieve that by installing [pysal/mgwr](#) by running the following from a command shell:

```
pip install https://github.com/pysal/mgwr/archive/master.zip
```

You can also [fork](#) the [pysal/mgwr](#) repo and create a local clone of your fork. By making changes to your local clone and submitting a pull request to [pysal/mgwr](#), you can contribute to the mgwr development.





## 2.1 GWR Model Estimation and Inference

<code>mgwr.gwr.GWR(coords, y, X, bw[, family, ...])</code>	Geographically weighted regression.
<code>mgwr.gwr.GWRResults(model, params, predy, S, CCT)</code>	Basic class including common properties for all GWR regression models
<code>mgwr.gwr.GWRResultsLite(model, resid, influ)</code>	Lightweight GWR that computes the minimum diagnostics needed for bandwidth selection

### 2.1.1 mgwr.gwr.GWR

**class** `mgwr.gwr.GWR`(*coords*, *y*, *X*, *bw*, *family*=<*spglm.family.Gaussian* object>, *offset*=None, *sigma2\_vl*=True, *kernel*='bisquare', *fixed*=False, *constant*=True, *dmat*=None, *sorted\_dmat*=None, *spherical*=False)

Geographically weighted regression. Can currently estimate Gaussian, Poisson, and logistic models(built on a GLM framework). GWR object prepares model input. Fit method performs estimation and returns a GWRResults object.

#### Parameters

**coords** [array-like] n\*2, collection of n sets of (x,y) coordinates of observatons; also used as calibration locations is 'points' is set to None

**y** [array] n\*1, dependent variable

**X** [array] n\*k, independent variable, exlcuding the constant

**bw** [scalar] bandwidth value consisting of either a distance or N nearest neighbors; user specified or obtained using Sel\_BW

**family** [family object] underlying probability model; provides distribution-specific calculations

**offset** [array] n\*1, the offset variable at the ith location. For Poisson model this term is often the size of the population at risk or the expected size of the outcome in spatial epidemiology Default is None where Ni becomes 1.0 for all locations; only for Poisson models

**sigma2\_v1** [boolean] specify form of corrected denominator of sigma squared to use for model diagnostics; Acceptable options are:

‘True’:  $n - \text{tr}(S)$  (default) ‘False’:  $n - 2(\text{tr}(S) + \text{tr}(S'S))$

**kernel** [string] type of kernel function used to weight observations; available options: ‘gaussian’ ‘bisquare’ ‘exponential’

**fixed** [boolean] True for distance based kernel function and False for adaptive (nearest neighbor) kernel function (default)

**constant** [boolean] True to include intercept (default) in model and False to exclude intercept.

**dmat** [array]  $n \times n$ , distance matrix between calibration locations used to compute weight matrix. Defaults to None and is primarily for avoiding duplicate computation during bandwidth selection.

**sorted\_dmat** [array]  $n \times n$ , sorted distance matrix between calibration locations used to compute weight matrix. Defaults to None and is primarily for avoiding duplicate computation during bandwidth selection.

**spherical** [boolean] True for spherical coordinates (long-lat), False for projected coordinates (default).

## Examples

#basic model calibration

```
>>> import libpysal as ps
>>> from mgwr.gwr import GWR
>>> data = ps.io.open(ps.examples.get_path('GData_utm.csv'))
>>> coords = list(zip(data.by_col('X'), data.by_col('Y')))
>>> y = np.array(data.by_col('PctBach')).reshape((-1,1))
>>> rural = np.array(data.by_col('PctRural')).reshape((-1,1))
>>> pov = np.array(data.by_col('PctPov')).reshape((-1,1))
>>> african_amer = np.array(data.by_col('PctBlack')).reshape((-1,1))
>>> X = np.hstack([rural, pov, african_amer])
>>> model = GWR(coords, y, X, bw=90.000, fixed=False, kernel='bisquare')
>>> results = model.fit()
>>> print(results.params.shape)
(159, 4)
```

#predict at unsampled locations

```
>>> index = np.arange(len(y))
>>> test = index[-10:]
>>> X_test = X[test]
>>> coords_test = np.array(coords)[test]
>>> model = GWR(coords, y, X, bw=94, fixed=False, kernel='bisquare')
>>> results = model.predict(coords_test, X_test)
>>> print(results.params.shape)
(10, 4)
```

## Attributes

**coords** [array-like]  $n \times 2$ , collection of  $n$  sets of (x,y) coordinates used for calibration locations

**y** [array]  $n \times 1$ , dependent variable

**X** [array]  $n \times k$ , independent variable, excluding the constant

**bw** [scalar] bandwidth value consisting of either a distance or N nearest neighbors; user specified or obtained using Sel\_BW

**family** [family object] underlying probability model; provides distribution-specific calculations

**offset** [array]  $n \times 1$ , the offset variable at the  $i$ th location. For Poisson model this term is often the size of the population at risk or the expected size of the outcome in spatial epidemiology. Default is None where  $N_i$  becomes 1.0 for all locations

**sigma2\_v1** [boolean] specify form of corrected denominator of sigma squared to use for model diagnostics; Acceptable options are:  
 ‘True’:  $n - \text{tr}(S)$  (default) ‘False’:  $n - 2(\text{tr}(S) + \text{tr}(S'S))$

**kernel** [string] type of kernel function used to weight observations; available options: ‘gaussian’ ‘bisquare’ ‘exponential’

**fixed** [boolean] True for distance based kernel function and False for adaptive (nearest neighbor) kernel function (default)

**constant** [boolean] True to include intercept (default) in model and False to exclude intercept

**dmat** [array]  $n \times n$ , distance matrix between calibration locations used to compute weight matrix. Defaults to None and is primarily for avoiding duplicate computation during bandwidth selection.

**sorted\_dmat** [array]  $n \times n$ , sorted distance matrix between calibration locations used to compute weight matrix. Defaults to None and is primarily for avoiding duplicate computation during bandwidth selection.

**spherical** [boolean] True for spherical coordinates (long-lat), False for projected coordinates (default).

**n** [integer] number of observations

**k** [integer] number of independent variables

**mean\_y** [float] mean of y

**std\_y** [float] standard deviation of y

**fit\_params** [dict] parameters passed into fit method to define estimation routine

**W** [array]  $n \times n$ , spatial weights matrix for weighting all observations from each calibration point

**points** [array-like]  $n \times 2$ , collection of n sets of (x,y) coordinates used for calibration locations instead of all observations; defaults to None unless specified in predict method

**P** [array]  $n \times k$ , independent variables used to make prediction; excluding the constant; default to None unless specified in predict method

**exog\_scale** [scalar] estimated scale using sampled locations; default is None unless specified in predict method

**exog\_resid** [array-like] estimated residuals using sampled locations; default is None unless specified in predict method

## Methods

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`fit([ini_params, tol, max_iter, solve, ...])`

Method that fits a model with a particular estimation routine.

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Table 2 – continued from previous page

<code>predict(points, P[, exog_scale, exog_resid, ...])</code>	Method that predicts values of the dependent variable at un-sampled locations
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**mgwr.gwr.GWR**

GWR.**fit** (*ini\_params=None, tol=1e-05, max\_iter=20, solve='iwls', searching=False*)

Method that fits a model with a particular estimation routine.

**Parameters**

**ini\_betas** [array, optional]  $k \times 1$ , initial coefficient values, including constant. Default is None, which calculates initial values during estimation.

**tol: float, optional** Tolerance for estimation convergence. Default is  $1.0e-5$ .

**max\_iter** [integer, optional] Maximum number of iterations if convergence not achieved. Default is 20.

**solve** [string, optional] Technique to solve MLE equations. Default is 'iwls', meaning iteratively (re)weighted least squares.

**searching** [bool, optional] Whether to estimate a lightweight GWR that computes the minimum diagnostics needed for bandwidth selection (could speed up bandwidth selection for GWR) or to estimate a full GWR. Default is False.

**Returns**

: If searching=True, return a GWRResult instance; otherwise, return a GWRResultLite instance.

**mgwr.gwr.GWR**

GWR.**predict** (*points, P, exog\_scale=None, exog\_resid=None, fit\_params={}*)

Method that predicts values of the dependent variable at un-sampled locations

**Parameters**

**points** [array-like]  $n \times 2$ , collection of  $n$  sets of (x,y) coordinates used for calibration prediction locations

**P** [array]  $n \times k$ , independent variables used to make prediction; excluding the constant

**exog\_scale** [scalar] estimated scale using sampled locations; default is None which estimates a model using points from "coords"

**exog\_resid** [array-like] estimated residuals using sampled locations; default is None which estimates a model using points from "coords"; if given it must be  $n \times 1$  where  $n$  is the length of coords

**fit\_params** [dict] key-value pairs of parameters that will be passed into fit method to define estimation routine; see fit method for more details

<b>df_model</b>	
<b>df_resid</b>	

**\_\_init\_\_** (*coords, y, X, bw, family=<spgml.family.Gaussian object>, offset=None, sigma2\_v1=True, kernel='bisquare', fixed=False, constant=True, dmat=None, sorted\_dmat=None, spherical=False*)

Initialize class

## 2.1.2 mgwr.gwr.GWRResults

**class** mgwr.gwr.GWRResults (*model, params, predy, S, CCT, w=None*)

Basic class including common properties for all GWR regression models

### Parameters

**model** [GWR object] pointer to GWR object with estimation parameters  
**params** [array]  $n \times k$ , estimated coefficients  
**predy** [array]  $n \times 1$ , predicted y values  
**S** [array]  $n \times n$ , hat matrix  
**CCT** [array]  $n \times k$ , scaled variance-covariance matrix  
**w** [array]  $n \times 1$ , final weight used for iteratively re-weighted least squares; default is None

### Attributes

**model** [GWR Object] points to GWR object for which parameters have been estimated  
**params** [array]  $n \times k$ , parameter estimates  
**predy** [array]  $n \times 1$ , predicted value of y  
**y** [array]  $n \times 1$ , dependent variable  
**X** [array]  $n \times k$ , independent variable, including constant  
**family** [family object] underlying probability model; provides distribution-specific calculations  
**n** [integer] number of observations  
**k** [integer] number of independent variables  
**df\_model** [integer] model degrees of freedom  
**df\_resid** [integer] residual degrees of freedom  
**offset** [array]  $n \times 1$ , the offset variable at the  $i$ th location. For Poisson model this term is often the size of the population at risk or the expected size of the outcome in spatial epidemiology; Default is None where  $N_i$  becomes 1.0 for all locations  
**scale** [float] sigma squared used for subsequent computations  
**w** [array]  $n \times 1$ , final weights from iteratively re-weighted least squares routine  
**resid\_response** [array]  $n \times 1$ , residuals of the response  
**resid\_ss** [scalar] residual sum of squares  
**W** [array]  $n \times n$ ; spatial weights for each observation from each calibration point  
**S** [array]  $n \times n$ , hat matrix  
**CCT** [array]  $n \times k$ , scaled variance-covariance matrix  
**ENP** [scalar] effective number of parameters  
**tr\_S** [float] trace of S (hat) matrix  
**tr\_STS** [float] trace of STS matrix  
**y\_bar** [array] weighted mean of y  
**TSS** [array] geographically weighted total sum of squares  
**RSS** [array] geographically weighted residual sum of squares

**R2** [float] R-squared for the entire model (1- RSS/TSS)

**aic** [float] Akaike information criterion

**aicc** [float] corrected Akaike information criterion to account to account for model complexity (smaller bandwidths)

**bic** [float] Bayesian information criterio

**localR2** [array] local R square

**sigma2** [float] residual variance

**std\_res** [array] standardized residuals

**bse** [array] standard errors of Betas

**infl** [array] Influence: leading diagonal of S Matrix

**CooksD** [array] n\*1, Cook's D

**tvalues** [array] Return the t-statistic for a given parameter estimate.

**adj\_alpha** [array] Corrected alpha (critical) values to account for multiple testing during hypothesis testing.

**deviance** [array] n\*1, local model deviance for each calibration point

**resid\_deviance** [array] n\*1, local sum of residual deviance for each calibration point

**llf** [scalar] log-likelihood of the full model; see `pysal.contrib.glm.family` for damily-sepcific log-likelihoods

**pDev** [float] Local percentage of deviance accounted for.

**mu** [array] n\*, flat one dimensional array of predicted mean response value from estimator

**fit\_params** [dict] parameters passed into fit method to define estimation routine

**predictions** [array] p\*1, predicted values generated by calling the GWR predict method to predict dependent variable at unsampled points ()

## Methods

<code>ENP()</code>	effective number of parameters
<code>RSS()</code>	geographically weighted residual sum of squares
<code>TSS()</code>	geographically weighted total sum of squares
<code>adj_alpha()</code>	Corrected alpha (critical) values to account for multiple testing during hypothesis testing.
<code>bse()</code>	standard errors of Betas
<code>conf_int()</code>	Returns the confidence interval of the fitted parameters.
<code>cooksD()</code>	Influence: leading diagonal of S Matrix
<code>cov_params(cov[, exog_scale])</code>	Returns scaled covariance parameters
<code>critical_tval([alpha])</code>	Utility function to derive the critial t-value based on given alpha that are needed for hypothesis testing
<code>filter_tvals([critical_t, alpha])</code>	Utility function to set tvalues with an absolute value smaller than the absolute value of the alpha (critical) value to 0.
<code>infl()</code>	Influence: leading diagonal of S Matrix

Continued on next page

Table 3 – continued from previous page

<code>localR2()</code>	local R square
<code>local_collinearity()</code>	Computes several indicators of multicollinearity within a geographically weighted design matrix, including:
<code>pDev()</code>	Local percentage of deviance accounted for.
<code>sigma2()</code>	residual variance
<code>spatial_variability(selector[, n_iters, seed])</code>	Method to compute a Monte Carlo test of spatial variability for each estimated coefficient surface.
<code>std_res()</code>	standardized residuals
<code>summary()</code>	Print out GWR summary
<code>tr_S()</code>	trace of S (hat) matrix
<code>tr_STS()</code>	trace of STS matrix
<code>tvalues()</code>	Return the t-statistic for a given parameter estimate.
<code>use_t()</code>	bool(x) -> bool
<code>y_bar()</code>	weighted mean of y

<b>D2</b>	
<b>R2</b>	
<b>adj_D2</b>	
<b>adj_pseudoR2</b>	
<b>aic</b>	
<b>aicc</b>	
<b>bic</b>	
<b>deviance</b>	
<b>df_model</b>	
<b>df_resid</b>	
<b>initialize</b>	
<b>llf</b>	
<b>llnull</b>	
<b>normalized_cov_params</b>	
<b>null</b>	
<b>null_deviance</b>	
<b>pearson_chi2</b>	
<b>predictions</b>	
<b>pseudoR2</b>	
<b>pvalues</b>	
<b>resid_anscombe</b>	
<b>resid_deviance</b>	
<b>resid_pearson</b>	
<b>resid_response</b>	
<b>resid_ss</b>	
<b>resid_working</b>	
<b>scale</b>	

`__init__` (*model, params, predy, S, CCT, w=None*)  
Initialize self. See `help(type(self))` for accurate signature.

### 2.1.3 mgwr.gwr.GWRResultsLite

**class** mgwr.gwr.**GWRResultsLite** (*model, resid, influ*)

Lightweight GWR that computes the minimum diagnostics needed for bandwidth selection

#### Parameters

**model** [GWR object] pointer to GWR object with estimation parameters

**resid** [array] n\*1, residuals of the repsonse

**influ** [array] n\*1, leading diagonal of S matrix

#### Attributes

**tr\_S** [float] trace of S (hat) matrix

**llf** [scalar] log-likelihood of the full model; see `pysal.contrib.glm.family` for damily-sepcific log-likelihoods

**mu** [array] n\*, flat one dimensional array of predicted mean response value from estimator

**resid\_ss** [scalar] residual sum of sqaures

#### Methods

<b>llf</b>	
<b>mu</b>	
<b>resid_ss</b>	
<b>tr_S</b>	

**\_\_init\_\_** (*model, resid, influ*)

Initialize self. See `help(type(self))` for accurate signature.

## 2.2 MGWR Estimation and Inference

---

<code>mgwr.gwr.MGWR</code> ( <i>coords, y, X, selector[, ...]</i> )	Multiscale GWR estimation and inference.
<code>mgwr.gwr.MGWRResults</code> ( <i>model, params, predy, ...</i> )	Class including common properties for a MGWR model.

---

### 2.2.1 mgwr.gwr.MGWR

**class** mgwr.gwr.**MGWR** (*coords, y, X, selector, sigma2\_v1=True, kernel='bisquare', fixed=False, constant=True, dmat=None, sorted\_dmat=None, spherical=False*)

Multiscale GWR estimation and inference.

#### Parameters

**coords** [array-like] n\*2, collection of n sets of (x,y) coordinates of observatons; also used as calibration locations is 'points' is set to None

**y** [array] n\*1, dependent variable

**X** [array] n\*k, independent variable, exlcuding the constant

**selector** [sel\_bw object] valid sel\_bw object that has successfully called the "search" method.



This parameter passes on information from GAM model estimation including optimal bandwidths.

**family** [family object] underlying probability model; provides distribution-specific calculations

**sigma2\_v1** [boolean] specify form of corrected denominator of sigma squared to use for model diagnostics; Acceptable options are:

‘True’:  $n - \text{tr}(S)$  (default) ‘False’:  $n - 2(\text{tr}(S) + \text{tr}(S'S))$

**kernel** [string] type of kernel function used to weight observations; available options: ‘gaussian’ ‘bisquare’ ‘exponential’

**fixed** [boolean] True for distance based kernel function and False for adaptive (nearest neighbor) kernel function (default)

**constant** [boolean] True to include intercept (default) in model and False to exclude intercept.

**dmat** [array]  $n \times n$ , distance matrix between calibration locations used to compute weight matrix. Defaults to None and is primarily for avoiding duplicate computation during bandwidth selection.

**sorted\_dmat** [array]  $n \times n$ , sorted distance matrix between calibration locations used to compute weight matrix. Defaults to None and is primarily for avoiding duplicate computation during bandwidth selection.

**spherical** [boolean] True for spherical coordinates (long-lat), False for projected coordinates (default).

## Examples

#basic model calibration

```
>>> import libpysal as ps
>>> from mgwr.gwr import MGWR
>>> from mgwr.sel_bw import Sel_BW
>>> data = ps.io.open(ps.examples.get_path('GData_utm.csv'))
>>> coords = list(zip(data.by_col('X'), data.by_col('Y')))
>>> y = np.array(data.by_col('PctBach')).reshape((-1,1))
>>> rural = np.array(data.by_col('PctRural')).reshape((-1,1))
>>> fb = np.array(data.by_col('PctFB')).reshape((-1,1))
>>> african_amer = np.array(data.by_col('PctBlack')).reshape((-1,1))
>>> X = np.hstack([fb, african_amer, rural])
>>> X = (X - X.mean(axis=0)) / X.std(axis=0)
>>> y = (y - y.mean(axis=0)) / y.std(axis=0)
>>> selector = Sel_BW(coords, y, X, multi=True)
>>> selector.search(multi_bw_min=[2])
[92.0, 101.0, 136.0, 158.0]
>>> model = MGWR(coords, y, X, selector, fixed=False, kernel='bisquare', sigma2_
↪v1=True)
>>> results = model.fit()
>>> print(results.params.shape)
(159, 4)
```

## Attributes

**coords** [array-like]  $n \times 2$ , collection of  $n$  sets of  $(x,y)$  coordinates of observatons; also used as calibration locations is ‘points’ is set to None

**y** [array]  $n \times 1$ , dependent variable

- X** [array]  $n \times k$ , independent variable, excluding the constant
- selector** [sel\_bw object] valid sel\_bw object that has successfully called the “search” method. This parameter passes on information from GAM model estimation including optimal bandwidths.
- bw** [array-like] collection of bandwidth values consisting of either a distance or N nearest neighbors; user specified or obtained using Sel\_BW with fb=True. Order of values should be the same as the order of columns associated with X
- family** [family object] underlying probability model; provides distribution-specific calculations
- sigma2\_v1** [boolean] specify form of corrected denominator of sigma squared to use for model diagnostics; Acceptable options are:  
‘True’:  $n - \text{tr}(S)$  (default) ‘False’:  $n - 2(\text{tr}(S) + \text{tr}(S'S))$
- kernel** [string] type of kernel function used to weight observations; available options: ‘gaussian’ ‘bisquare’ ‘exponential’
- fixed** [boolean] True for distance based kernel function and False for adaptive (nearest neighbor) kernel function (default)
- constant** [boolean] True to include intercept (default) in model and False to exclude intercept.
- dmat** [array]  $n \times n$ , distance matrix between calibration locations used to compute weight matrix. Defaults to None and is primarily for avoiding duplicate computation during bandwidth selection.
- sorted\_dmat** [array]  $n \times n$ , sorted distance matrix between calibration locations used to compute weight matrix. Defaults to None and is primarily for avoiding duplicate computation during bandwidth selection.
- spherical** [boolean] True for spherical coordinates (long-lat), False for projected coordinates (default).
- n** [integer] number of observations
- k** [integer] number of independent variables
- mean\_y** [float] mean of y
- std\_y** [float] standard deviation of y
- fit\_params** [dict] parameters passed into fit method to define estimation routine
- W** [array-like] list of  $n \times n$  arrays, spatial weights matrices for weighting all observations from each calibration point: one for each covariate (k)

## Methods

---

<code>fit()</code>	Method that extracts information from Sel_BW (selector) object and prepares GAM estimation results for MGWRResults object.
<code>predict()</code>	Not implemented.

---

## mgwr.gwr.MGWR

`MGWR.fit()`

Method that extracts information from Sel\_BW (selector) object and prepares GAM estimation results for MGWRResults object.

**mgwr.gwr.MGWR**

**MGWR.predict()**  
Not implemented.

<b>df_model</b>	
<b>df_resid</b>	

**\_\_init\_\_**(*coords, y, X, selector, sigma2\_vl=True, kernel='bisquare', fixed=False, constant=True, dmat=None, sorted\_dmat=None, spherical=False*)  
Initialize class

**2.2.2 mgwr.gwr.MGWRResults**

**class mgwr.gwr.MGWRResults**(*model, params, predy, S, CCT, R, w*)  
Class including common properties for a MGWR model.

**Parameters**

**model** [MGWR object] pointer to MGWR object with estimation parameters  
**params** [array] n\*k, estimated coefficients  
**predy** [array] n\*1, predicted y values  
**S** [array] n\*n, hat matrix  
**R** [array] n\*n\*k, partial hat matrices for each covariate  
**CCT** [array] n\*k, scaled variance-covariance matrix  
**w** [array] n\*1, final weight used for iteratively re-weighted least squares; default is None

**Attributes**

**model** [GWR Object] points to GWR object for which parameters have been estimated  
**params** [array] n\*k, parameter estimates  
**predy** [array] n\*1, predicted value of y  
**y** [array] n\*1, dependent variable  
**X** [array] n\*k, independent variable, including constant  
**family** [family object] underlying probability model; provides distribution-specific calculations  
**n** [integer] number of observations  
**k** [integer] number of independent variables  
**df\_model** [integer] model degrees of freedom  
**df\_resid** [integer] residual degrees of freedom  
**scale** [float] sigma squared used for subsequent computations  
**w** [array] n\*1, final weights from iteratively re-weighted least squares routine  
**resid\_response** [array] n\*1, residuals of the response  
**resid\_ss** [scalar] residual sum of squares  
**W** [array-like] list of n\*n arrays, spatial weights matrices for weighting all observations from each calibration point: one for each covariate (k)

**S** [array]  $n \times n$ , hat matrix

**R** [array]  $n \times n \times k$ , partial hat matrices for each covariate

**CCT** [array]  $n \times k$ , scaled variance-covariance matrix

**ENP** [scalar] effective number of parameters

**ENP\_j** [array-like] effective number of parameters, which depends on `sigma2`, for each covariate in the model

**adj\_alpha** [array] Corrected alpha (critical) values to account for multiple testing during hypothesis testing.

**adj\_alpha\_j** [array] Corrected alpha (critical) values to account for multiple testing during hypothesis testing.

**tr\_S** [float] trace of **S** (hat) matrix

**tr\_STS** [float] trace of **STS** matrix

**R2** [float] R-squared for the entire model (1- `RSS/TSS`)

**aic** [float] Akaike information criterion

**aicc** [float] corrected Akaike information criterion to account to account for model complexity (smaller bandwidths)

**bic** [float] Bayesian information criterion

**sigma2** [float] residual variance

**std\_res** [array] standardized residuals

**bse** [array] standard errors of Betas

**infl** [array] Influence: leading diagonal of **S** Matrix

**CooksD** [array]  $n \times 1$ , Cook's D

**tvalues** [array] Return the t-statistic for a given parameter estimate.

**llf** [scalar] log-likelihood of the full model; see `pysal.contrib.glm.family` for family-specific log-likelihoods

**mu** [array]  $n \times 1$ , flat one dimensional array of predicted mean response value from estimator

## Methods

<code>ENP()</code>	effective number of parameters
<code>RSS()</code>	geographically weighted residual sum of squares
<code>TSS()</code>	geographically weighted total sum of squares
<code>adj_alpha()</code>	Corrected alpha (critical) values to account for multiple testing during hypothesis testing.
<code>adj_alpha_j()</code>	Corrected alpha (critical) values to account for multiple testing during hypothesis testing.
<code>bse()</code>	standard errors of Betas
<code>conf_int()</code>	Returns the confidence interval of the fitted parameters.
<code>cooksD()</code>	Influence: leading diagonal of <b>S</b> Matrix
<code>cov_params(cov[, exog_scale])</code>	Returns scaled covariance parameters

Continued on next page

Table 6 – continued from previous page

<code>critical_tval([alpha])</code>	Utility function to derive the critical t-value based on given alpha that are needed for hypothesis testing
<code>filter_tvals([critical_t, alpha])</code>	Utility function to set tvalues with an absolute value smaller than the absolute value of the alpha (critical) value to 0.
<code>influ()</code>	Influence: leading diagonal of S Matrix
<code>localR2()</code>	local R square
<code>local_collinearity()</code>	Computes several indicators of multicollinearity within a geographically weighted design matrix, including:
<code>pDev()</code>	Local percentage of deviance accounted for.
<code>sigma2()</code>	residual variance
<code>spatial_variability(selector[, n_iters, seed])</code>	Method to compute a Monte Carlo test of spatial variability for each estimated coefficient surface.
<code>std_res()</code>	standardized residuals
<code>summary()</code>	Print out MGWR summary
<code>tr_S()</code>	trace of S (hat) matrix
<code>tr_STS()</code>	trace of STS matrix
<code>tvalues()</code>	Return the t-statistic for a given parameter estimate.
<code>use_t()</code>	bool(x) -> bool
<code>y_bar()</code>	weighted mean of y

**mgwr.gwr.MGWRResults**`MGWRResults.critical_tval(alpha=None)`

Utility function to derive the critical t-value based on given alpha that are needed for hypothesis testing

**Parameters**

**alpha** [scalar] critical value to determine which tvalues are associated with statistically significant parameter estimates. Default to None in which case the adjusted alpha value at the 95 percent CI is automatically used.

**Returns**

**critical** [scalar] critical t-val based on alpha

**mgwr.gwr.MGWRResults**`MGWRResults.filter_tvals(critical_t=None, alpha=None)`

Utility function to set tvalues with an absolute value smaller than the absolute value of the alpha (critical) value to 0. If critical\_t is supplied than it is used directly to filter. If alpha is provided than the critical t value will be derived and used to filter. If neither are critical\_t nor alpha are provided, an adjusted alpha at the 95 percent CI will automatically be used to define the critical t-value and used to filter. If both critical\_t and alpha are supplied then the alpha value will be ignored.

**Parameters**

**critical** [scalar] critical t-value to determine whether parameters are statistically significant

**alpha** [scalar] alpha value to determine which tvalues are associated with statistically significant parameter estimates

**Returns**

**filtered** [array]  $n \times k$ ; new set of  $n$  tvalues for each of  $k$  variables where absolute tvalues less than the absolute value of alpha have been set to 0.

### mgwr.gwr.MGWRResults

`MGWRResults.local_collinearity()`

Computes several indicators of multicollinearity within a geographically weighted design matrix, including:

local condition number ( $n, 1$ ) local variance-decomposition proportions ( $n, p$ )

Returns four arrays with the order and dimensions listed above where  $n$  is the number of locations used as calibrations points and  $p$  is the nubmer of explanatory variables

### mgwr.gwr.MGWRResults

`MGWRResults.spatial_variability(selector, n_iters=1000, seed=None)`

Method to compute a Monte Carlo test of spatial variability for each estimated coefficient surface.

WARNING: This test is very computationally demanding!

#### Parameters

**selector** [sel\_bw object] should be the sel\_bw object used to select a bandwidth for the gwr model that produced the surfaces that are being tested for spatial variation

**n\_iters** [int] the number of Monte Carlo iterations to include for the tests of spatial variability.

**seed** [int] optional parameter to select a custom seed to ensure stochastic results are replicable. Default is none which automatically sets the seed to 5536

#### Returns

**p values** [list] a list of psuedo p-values that correspond to the model parameter surfaces. Allows us to assess the probability of obtaining the observed spatial variation of a given surface by random chance.

### mgwr.gwr.MGWRResults

`MGWRResults.summary()`

Print out MGWR summary

D2	
ENP_j	
R2	
adj_D2	
adj_pseudoR2	
aic	
aicc	
bic	
deviance	
df_model	
df_resid	
initialize	
llf	
llnull	
normalized_cov_params	
null	
null_deviance	
pearson_chi2	
predictions	
pseudoR2	
pvalues	
resid_anscombe	
resid_deviance	
resid_pearson	
resid_response	
resid_ss	
resid_working	
scale	

`__init__` (*model, params, predy, S, CCT, R, w*)  
Initialize class

## 2.3 Utility Functions

### 2.3.1 Kernel Specification

<code>mgwr.kernels.fix_gauss(coords, bw[, points, ...])</code>	Fixed Gaussian kernel.
<code>mgwr.kernels.adapt_gauss(coords, nn[, ...])</code>	Spatially adaptive Gaussian kernel.
<code>mgwr.kernels.fix_bisquare(coords, bw[, ...])</code>	Fixed bisquare kernel.
<code>mgwr.kernels.adapt_bisquare(coords, nn[, ...])</code>	Spatially adaptive bisquare kernel.
<code>mgwr.kernels.fix_exp(coords, bw[, points, ...])</code>	Fixed exponential kernel.
<code>mgwr.kernels.adapt_exp(coords, nn[, points, ...])</code>	Spatially adaptive exponential kernel.

### mgwr.kernels.fix\_gauss

`mgwr.kernels.fix_gauss` (*coords*, *bw*, *points=None*, *dmat=None*, *sorted\_dmat=None*, *spherical=False*)  
Fixed Gaussian kernel.

### mgwr.kernels.adapt\_gauss

`mgwr.kernels.adapt_gauss` (*coords*, *nn*, *points=None*, *dmat=None*, *sorted\_dmat=None*, *spherical=False*)  
Spatially adaptive Gaussian kernel.

### mgwr.kernels.fix\_bisquare

`mgwr.kernels.fix_bisquare` (*coords*, *bw*, *points=None*, *dmat=None*, *sorted\_dmat=None*, *spherical=False*)  
Fixed bisquare kernel.

### mgwr.kernels.adapt\_bisquare

`mgwr.kernels.adapt_bisquare` (*coords*, *nn*, *points=None*, *dmat=None*, *sorted\_dmat=None*, *spherical=False*)  
Spatially adaptive bisquare kernel.

### mgwr.kernels.fix\_exp

`mgwr.kernels.fix_exp` (*coords*, *bw*, *points=None*, *dmat=None*, *sorted\_dmat=None*, *spherical=False*)  
Fixed exponential kernel.

### mgwr.kernels.adapt\_exp

`mgwr.kernels.adapt_exp` (*coords*, *nn*, *points=None*, *dmat=None*, *sorted\_dmat=None*, *spherical=False*)  
Spatially adaptive exponential kernel.

## 2.3.2 Bandwidth Selection

---

<code>mgwr.sel_bw.Sel_BW</code> ( <i>coords</i> , <i>y</i> , <i>X_loc</i> [...])	Select bandwidth for kernel
--	-----------------------------

---

### mgwr.sel\_bw.Sel\_BW

**class** `mgwr.sel_bw.Sel_BW` (*coords*, *y*, *X\_loc*, *X\_glob=None*, *family=<spglm.family.Gaussian object>*, *offset=None*, *kernel='bisquare'*, *fixed=False*, *multi=False*, *constant=True*, *spherical=False*)

Select bandwidth for kernel

Methods: p211 - p213, bandwidth selection Fotheringham, A. S., Brunsdon, C., & Charlton, M. (2002). Geographically weighted regression: the analysis of spatially varying relationships.

#### Parameters



**y** [array] n\*1, dependent variable.

**X\_glob** [array] n\*k1, fixed independent variable.

**X\_loc** [array] n\*k2, local independent variable, including constant.

**coords** [list of tuples] (x,y) of points used in bandwidth selection

**family** [string] GWR model type: 'Gaussian', 'logistic', 'Poisson'

**offset** [array] n\*1, the offset variable at the ith location. For Poisson model this term is often the size of the population at risk or the expected size of the outcome in spatial epidemiology  
Default is None where  $N_i$  becomes 1.0 for all locations

**kernel** [string] kernel function: 'gaussian', 'bisquare', 'exponential'

**fixed** [boolean] True for fixed bandwidth and False for adaptive (NN)

**multi** [True for multiple (covariate-specific) bandwidths] False for a traditional (same for all covariates) bandwidth; default is False.

**constant** [boolean] True to include intercept (default) in model and False to exclude intercept.

**spherical** [boolean] True for spherical coordinates (long-lat), False for projected coordinates (default).

## Examples

```
>>> import libpysal as ps
>>> from mgwr.sel_bw import Sel_BW
>>> data = ps.io.open(ps.examples.get_path('GData_utm.csv'))
>>> coords = list(zip(data.by_col('X'), data.by_col('Y')))
>>> y = np.array(data.by_col('PctBach')).reshape((-1,1))
>>> rural = np.array(data.by_col('PctRural')).reshape((-1,1))
>>> pov = np.array(data.by_col('PctPov')).reshape((-1,1))
>>> african_amer = np.array(data.by_col('PctBlack')).reshape((-1,1))
>>> X = np.hstack([rural, pov, african_amer])
```

Golden section search AICc - adaptive bisquare

```
>>> bw = Sel_BW(coords, y, X).search(criterion='AICc')
>>> print(bw)
93.0
```

Golden section search AIC - adaptive Gaussian

```
>>> bw = Sel_BW(coords, y, X, kernel='gaussian').search(criterion='AIC')
>>> print(bw)
50.0
```

Golden section search BIC - adaptive Gaussian

```
>>> bw = Sel_BW(coords, y, X, kernel='gaussian').search(criterion='BIC')
>>> print(bw)
62.0
```

Golden section search CV - adaptive Gaussian

```
>>> bw = Sel_BW(coords, y, X, kernel='gaussian').search(criterion='CV')
>>> print(bw)
68.0
```

Interval AICc - fixed bisquare

```
>>> sel = Sel_BW(coords, y, X, fixed=True)
>>> bw = sel.search(search_method='interval', bw_min=211001.0, bw_max=211035.0,
↪ interval=2)
>>> print(bw)
211025.0
```

### Attributes

- y** [array] n\*1, dependent variable.
- X\_glob** [array] n\*k1, fixed independent variable.
- X\_loc** [array] n\*k2, local independent variable, including constant.
- coords** [list of tuples] (x,y) of points used in bandwidth selection
- family** [string] GWR model type: 'Gaussian', 'logistic', 'Poisson'
- kernel** [string] type of kernel used and whether fixed or adaptive
- fixed** [boolean] True for fixed bandwidth and False for adaptive (NN)
- criterion** [string] bw selection criterion: 'AICc', 'AIC', 'BIC', 'CV'
- search\_method** [string] bw search method: 'golden', 'interval'
- bw\_min** [float] min value used in bandwidth search
- bw\_max** [float] max value used in bandwidth search
- interval** [float] interval increment used in interval search
- tol** [float] tolerance used to determine convergence
- max\_iter** [integer] max iterations if no convergence to tol
- multi** [True for multiple (covariate-specific) bandwidths] False for a traditional (same for all covariates) bandwidth; default is False.
- constant** [boolean] True to include intercept (default) in model and False to exclude intercept.
- offset** [array] n\*1, the offset variable at the ith location. For Poisson model this term is often the size of the population at risk or the expected size of the outcome in spatial epidemiology. Default is None where  $N_i$  becomes 1.0 for all locations
- dmat** [array] n\*n, distance matrix between calibration locations used to compute weight matrix
- sorted\_dmat** [array] n\*n, sorted distance matrix between calibration locations used to compute weight matrix. Will be None for fixed bandwidths
- spherical** [boolean] True for spherical coordinates (long-lat), False for projected coordinates (default).
- search\_params** [dict] stores search arguments
- int\_score** [boolean] True if adaptive bandwidth is being used and bandwidth selection should be discrete. False if fixed bandwidth is being used and bandwidth does not have to be discrete.

- bw** [scalar or array-like] Derived optimal bandwidth(s). Will be a scalar for GWR (multi=False) and a list of scalars for MGWR (multi=True) with one bandwidth for each covariate.
- S** [array]  $n \times n$ , hat matrix derived from the iterative backfitting algorithm for MGWR during bandwidth selection
- R** [array]  $n \times n \times k$ , partial hat matrices derived from the iterative backfitting algorithm for MGWR during bandwidth selection. There is one  $n \times n$  matrix for each of the  $k$  covariates.
- params** [array]  $n \times k$ , calibrated parameter estimates for MGWR based on the iterative backfitting algorithm - computed and saved here to avoid having to do it again in the MGWR object.

## Methods

---

<code>search([search_method, criterion, bw_min, ...])</code>	Method to select one unique bandwidth for a gwr model or a bandwidth vector for a mgwr model.
--	---

---

## mgwr.sel\_bw.Sel\_BW

`Sel_BW.search` (*search\_method*='golden\_section', *criterion*='AICc', *bw\_min*=None, *bw\_max*=None, *interval*=0.0, *tol*=1e-06, *max\_iter*=200, *init\_multi*=None, *tol\_multi*=1e-05, *rss\_score*=False, *max\_iter\_multi*=200, *multi\_bw\_min*=[None], *multi\_bw\_max*=[None])

Method to select one unique bandwidth for a gwr model or a bandwidth vector for a mgwr model.

### Parameters

- criterion** [string] bw selection criterion: 'AICc', 'AIC', 'BIC', 'CV'
- search\_method** [string] bw search method: 'golden', 'interval'
- bw\_min** [float] min value used in bandwidth search
- bw\_max** [float] max value used in bandwidth search
- multi\_bw\_min** [list] min values used for each covariate in mgwr bandwidth search. Must be either a single value or have one value for each covariate including the intercept
- multi\_bw\_max** [list] max values used for each covariate in mgwr bandwidth search. Must be either a single value or have one value for each covariate including the intercept
- interval** [float] interval increment used in interval search
- tol** [float] tolerance used to determine convergence
- max\_iter** [integer] max iterations if no convergence to tol
- init\_multi** [float] None (default) to initialize MGWR with a bandwidth derived from GWR. Otherwise this option will choose the bandwidth to initialize MGWR with.
- tol\_multi** [convergence tolerance for the multiple bandwidth] backfitting algorithm; a larger tolerance may stop the algorithm faster though it may result in a less optimal model
- max\_iter\_multi** [max iterations if no convergence to tol for multiple] bandwidth backfittign algorithm
- rss\_score** [True to use the residual sum of sqaures to evaluate] each iteration of the multiple bandwidth backfitting routine and False to use a smooth function; default is False

### Returns

**bw** [scalar or array] optimal bandwidth value or values; returns scalar for multi=False and array for multi=True; ordering of bandwidths matches the ordering of the covariates (columns) of the designs matrix, X

**\_\_init\_\_** (*coords*, *y*, *X\_loc*, *X\_glob=None*, *family=<spglm.family.Gaussian object>*, *offset=None*, *kernel='bisquare'*, *fixed=False*, *multi=False*, *constant=True*, *spherical=False*)  
Initialize self. See help(type(self)) for accurate signature.

### 2.3.3 Visualization

---

<code>utils.shift_colormap(cmap[, start, ...])</code>	Function to offset the “center” of a colormap.
<code>utils.truncate_colormap(cmap[, minval, ...])</code>	Function to truncate a colormap by selecting a subset of the original colormap’s values
<code>utils.compare_surfaces(*args, **kwargs)</code>	

---

#### mgwr.utils.shift\_colormap

`mgwr.utils.shift_colormap(cmap, start=0, midpoint=0.5, stop=1.0, name='shiftedcmap')`

Function to offset the “center” of a colormap. Useful for data with a negative min and positive max and you want the middle of the colormap’s dynamic range to be at zero

##### Parameters

**cmap** [The matplotlib colormap to be altered]

**start** [Offset from lowest point in the colormap’s range.] Defaults to 0.0 (no lower offset). Should be between 0.0 and *midpoint*.

**midpoint** [The new center of the colormap. Defaults to] 0.5 (no shift). Should be between 0.0 and 1.0. In general, this should be  $1 - \text{vmax}/(\text{vmax} + \text{abs}(\text{vmin}))$  For example if your data range from -15.0 to +5.0 and you want the center of the colormap at 0.0, *midpoint* should be set to  $1 - 5/(5 + 15)$  or 0.75

**stop** [Offset from highets point in the colormap’s range.] Defaults to 1.0 (no upper offset). Should be between *midpoint* and 1.0.

##### Returns

**new\_cmap** [A new colormap that has been shifted.]

#### mgwr.utils.truncate\_colormap

`mgwr.utils.truncate_colormap(cmap, minval=0.0, maxval=1.0, n=100)`

Function to truncate a colormap by selecting a subset of the original colormap’s values

##### Parameters

**cmap** [Mmatplotlib colormap to be altered]

**minval** [Minimum value of the original colormap to include in the truncated colormap]

**maxval** [Maximum value of the original colormap to include in the truncated colormap]

**n** [Number of intervals between the min and max values for the gradient of the truncated colormap]

##### Returns

**new\_cmap** [A new colormap that has been shifted.]

### **mgwr.utils.compare\_surfaces**

`mgwr.utils.compare_surfaces(*args, **kwargs)`



## CHAPTER 3

---

### References

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